UNIT 2

Biased Bingo

What bingo card would give me the best chance of winning?

Materials

- Resource sheets as required
- Standard bingo cards & numbers (1–75)
- Counters (optional)
- 2 × 10-sided dice (1–10)

Mathematical Focus

- Number and Algebra – Maths facts (addition)
- Statistics and Probability – Data (frequency tables); Probability (likelihood of events, frequency charts)

Resource Sheets

- Resource sheet 1: Bingo Card Blanks
- Resource sheet 2: Numbers for Calling Bingo
- Resource sheet 3: Addition Table
- Resource sheet 4: Addition Table (Filled)
- Resource sheet 5: Addition Calling Cards
- Resource sheet 6: Likely Occurrence of Addition Facts

Support Website

www.curriculumpress.edu.au/maths

All of the resource sheets are available to download as PDF files. Those that you might customise are also available in Word document format.

WHAT HAPPENS?

In this unit, students devise bingo cards which they believe will give them the best chance of winning a game of addition fact bingo.

Students will:

- Familiarise themselves with the game of bingo. (Discover)
- Use their understanding about addition facts to identify which numbers will not occur (according to the game's rules). (Discover, Devise)
- Record the frequency with which the other numbers are expected to appear. (Devise)
- Construct a bingo card which they feel has the greatest likelihood of winning a game. (Develop)
- Test and reflect upon their decisions. (Defend)

TEACHER NOTES:

This activity requires the students to examine and identify patterns within maths. Not only does this familiarise them with their addition tables, it also gives them opportunities to construct frequency tables and to make decisions based upon the information that they gather. Finally, the element of chance in the game allows for further discussions on probability. Students find this activity interesting and enjoy the mild element of competition.

Student misconceptions about probability may surface. It is difficult as a teacher to refrain from correcting these misunderstandings, however in this unit, the activities are stepped in such a way that students will be able to identify many of their mistakes as they progress. This method of learning is particularly powerful and will foster deeper understanding.

BIASED BINGO

INACTION!

THE MATHEMATICS BEHIND THIS UNIT

Biased Bingo was designed to explore probability beyond coin tossing and dice rolling, two activities that have equally likely outcomes. Patterns in addition and multiplication tables do not conform so neatly and they provide a familiar context for students to explore outcomes that are not equally probable. As a result, students' misconceptions about probability become clearer and can be challenged.

RANDOMNESS AND PROBABILITY

Randomness describes a repeated process in which the outcomes cannot be pre-determined but rather follow a probability distribution. The theoretical probability of a certain outcome can be calculated. For example, if you were to roll a standard die, you would not expect it to roll in a consistent pattern. However you would expect that about one in six times you would roll a 6. Therefore, the probability of getting a six is 1 in 6 (1/6 or 0.167). To illustrate, the theoretical probability distribution for a dice throw is represented in Diagram 1. As you can see, each number is equally likely to occur.

By contrast, with the answers to the addition facts (using the numbers 1 through 10), there are some numbers that have 0 probability of being drawn, such as '1' (because the smallest fact is 1 + 1), or '21' (because the largest fact is 10 + 10). Some numbers, such as 5, have numerous chances of being drawn (4 chances: 1 + 4; 2 + 3; 3 + 2; 4 + 1). The probability distribution for the drawing of an answer to these number facts is represented in Diagram 2.

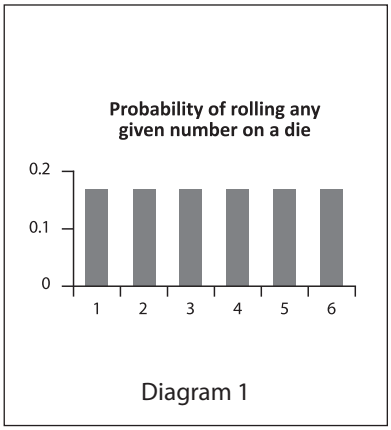


Diagram 1

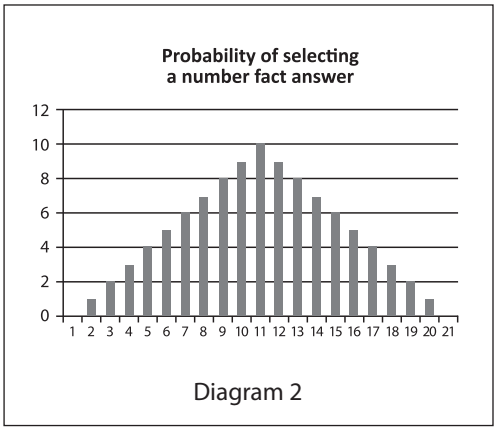


Diagram 2

Unlike our die, where every digit had the same chance of occurring, you can see that there are ten ways to achieve a sum equal to 11, and only one way to achieve a 2. If you were to draw a random number, there would be a much greater chance of the number being a 10, 11 or 12 than a 2 or a 20.

This is a difficult concept for children to understand. Very often they will have beliefs that surround 'luck' or 'sense'. For example, if a person needs a '6' to win a game, they will often argue that it is more likely or less likely based on whether they feel lucky or not. Students will often argue that if they have rolled a '1', for example, five times in a row that the next number is less likely to be '1', whereas in fact, every separate roll is a separate event and is equally likely to be any number regardless of what has just been rolled.

To develop these understandings fully, students need multiple opportunities to play with the language of probability and to construct frequency charts themselves. Resource sheet 4 shows the addition tables filled. It becomes much easier for students to envisage the outcomes if they construct a chart and count themselves the number of times each number can be rolled.

PLAY A GAME OF BINGO

Provide each student with a standard 5 × 5 bingo card, containing random numbers between 1 and 50 (for example). Do not use a multiplication or addition set. Calling each number only once, have students race to cover a full card.

These initial games of bingo will:
1. Give all students a common understanding of the rules of bingo.
2. Highlight the differences that emerge in the next rounds of addition bingo.

Commercial bingo sets are readily available in stores or online. Alternatively, give students a copy of Resource sheet 1 and have them select their own numbers. Reducing numbers to 1–36 will shorten the playing time.

DISCUSS

Engage the class in a discussion. The following questions would be useful to initiate debate:

- Whose card won?
- Do you think that card will win next time you play?
- Are there cards in the room that are more likely to win than other cards?
- Which cards are more likely to win? Why?

TIP

At this stage, elicit students' ideas but do not attempt to correct misconceptions; rather note them onto a projector or poster paper to review later.

REPLAY GAME

This time, as the numbers are being called, engage students in discussion by asking questions during the calling process.

Key Vocabulary

- Chance
- Probable/Probability
- Likelihood
- Certain/Certainty
- Possible/Possibility
- Impossible

You might write these words on the board or on a large sheet of paper as they come up. Keep the list in a prominent place for students to use during the unit to strengthen their mathematical language.

Key questions could include:

- What happens if you miss a number? Can you still win?
- Is it probable that the next number will be 7 (for example)? What is the likelihood of the next number being … (say the number just called – it would be impossible).
- How likely is it that the next number would be even/odd? (Usually the same, unless there are very few numbers left to call).
- Do you think the next number is more likely to be higher or lower (than the previous number)?

TIP

You may begin to see some common beliefs about randomness. For example:

> if the last three numbers drawn are odd, then it's 'time' for an even number, even though the likelihood, with so many numbers involved, is usually still fairly close to equal.

> the number they are waiting for is less likely/more likely than others depending on whether they are 'lucky' or not.

EXPERIMENT WITH GAME CARDS

Tell students that they are going to play a new game of bingo, but this time, it will be 'Addition Bingo'. In Addition Bingo, all of the numbers pulled out will be in the form of addition facts from 1+1 to 10+10. For example, 3 + 5 may be called. If students have 8 as one of their numbers, they can cover it (or cross it off).

Allowing students to use calculators or addition tables (Resource sheet 4) is an option. This activity is not meant to drill students on number facts. The focus is on number patterns and another way of thinking about addition facts, without limiting involvement of students with weaker skills.

- Provide all students with a copy of Resource sheet 1 and instruct them to choose numbers for their own bingo card for a game of Addition Bingo using addition facts from 1+1 to 10+10 (see Tip).
- Play a game using the calling cards provided in Resource sheet 5.

TIP

Students are unlikely to choose their numbers wisely at this stage. They may ask if they can use the same number more than once or use numbers that cannot be called, such as 0, 1 or 21. Do not try to correct their choices. They will usually realise part way through the game and if not, discussion at the end of the game will prompt these thoughts.

DISCUSSION

After the game ask the students to justify their choice of numbers. Key questions could include:

- Why did the card belonging to (name of student) win? Be sure to follow through on any comments that do not attribute the results purely to luck.

Assessment Ideas

Focused observation:

- Consider whether students are being drawn into the activity or not. Are they making suggestions? Are they considering others' suggestions? These can indicate students' development of necessary social/communication skills.
- It is useful to note the conceptions of randomness and number sense issues that arise – they provide a baseline or 'before' knowledge.

Resource sheet 5 includes a set of 100 calling cards and covers mathematics facts from 1–10, including turnarounds but excluding the 0 facts.

Assessment Ideas

Focused observation:

During discussion, determine which students are recognising that although luck plays a part, there are numbers that are more likely to be drawn and numbers that never occur.

Before moving on, ask students to write down strategies for winning, in their maths journals, as a reflection of their learning so far. This can be used as an assessment later to help you recognise if students are:

- Realising that some numbers cannot occur.
- Observing that some sums have a higher frequency (11 occurs ten times but 3 only occurs twice). The high frequency numbers therefore would be good choices – and can even appear more than once on the card.
- Able to justify their strategies mathematically and with mathematical language.
- Holding onto any unproductive beliefs about randomness (13 can't occur because it is an unlucky number).

- Do you think the same card would win next time? Why or why not?
- Which numbers on your card were never called (obtain these from students and write the list on the board)? Why do you think they didn't appear?

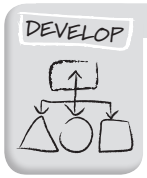
In groups, have students consider some strategies that they could use to create a game card most likely to win. Students should note some of the strategies and ideas in the comments section alongside their first bingo card on Resource sheet 1.

EXPERIMENT WITH GAME CARDS

Ask students to work in pairs (or individually) to create a card with a better chance of winning. Play again and have a discussion similar to the one above. Ask questions like:

- How could you change a card to make sure it couldn't win? (For example, putting a number twice that could only ever be called once or putting a number that will never be called, such as 0 or 1.)
- Is there any way that you could improve your chances of winning?
- Are specific numbers more likely to occur than others? (for example, those with more potential pairs, such as 11: 1+10, 2+9, 3+8, 4+7, 5+6, 6+5, 7+4, 8+3, 9+2, 10+1 as distinct from 2: 1+1 only).

- Provide students with a blank addition table grid (Resource sheet 3) to enable them to identify the frequency with which each sum occurs.
- Give them time with the problem to enable them to determine their own approaches and to see what understandings they are able to develop on their own.
- When students begin to stall, or if they are having difficulty understanding the task or how to approach it, draw the class back together to share their approaches. Ask the students:
  - Are there numbers that have a much higher probability of occurring?
  - Are there numbers that are much less likely to occur?
- Have students explain why they think their card has the 'best' chance of winning. Direct students' attention to the patterns in Resource sheet 3. Ask them 'How do you think you can use this information?' Have them respond individually in writing, to ascertain their level of understanding.
- Encourage students to predict whose game card will win and why.
- Play a game of bingo all the way through, using the calling cards provided in Resource sheet 5.
- Afterwards, discuss with the class which card won. Is it likely that the same card will win again next time? And the time after? Draw out the fact that while probability plays an important part, so does randomness. It is less likely that 2 and 20 will be drawn before 11 but it is not impossible.



DEVELOPING FAIR GAME CARDS

Challenge students to develop a set of four to six bingo cards for their group, each of which has an equal chance of winning. When discussing the activity, ensure they realise that simply rearranging the same numbers on the cards will not work as all cards would be covered at the same time.

TIP

Try bringing the class together to discuss their ideas about what 'fair' might mean. Students will need to consider the frequency with which each number is likely to occur. It is preferable that students determine this for themselves.

Use Resource sheet 6 to identify the frequency with which each number will occur (out of 100). Involve students in a discussion, asking questions such as:

- Are there numbers that have the same probability of occurring, such as 6 and 16?
- How could students identify other numbers with equal probability of occurring?
- How do they think they can use this information?

Assessment Idea

Task analysis:

- Ask students to each develop their own card within a group process so that you are able to fully determine each individual's level of understanding.

Copies of blank and filled addition tables are provided as Resource sheet 3 and Resource sheet 4 if required.

PRESENTATION

Have the student groups present their finished cards, justifying why they believe each card has an equal chance of winning. Students need to share the processes they used (including difficulties they encountered and strategies they discovered).

Assessment Idea

Task analysis:

- Did each group's set of cards contain numbers with equal probability of winning?

TO EXTEND

- In groups, have students compare the chances of each group's cards winning.
- Have students construct cards that could, but are unlikely to, win. Students can then compete to be the last one to call bingo.
- Extend the addition facts used, to: 1 to 12, 1 to 20, or 0 to 10.

TO SIMPLIFY

- Provide copies of filled addition tables (Resource sheet 4) to assist students who have not yet developed fluency with maths facts.
- Reduce the maths facts to addition of digits up to 5 or to multiplication facts recently covered.
- Use only the numbers 1 to 6.

ALTERNATIVE INQUIRIES OR ASSESSMENT

- Challenge students to design a card independently or in pairs for a game of Multiplication Bingo. What about Subtraction Bingo?

Having each student present a written justification of the process they followed along with the finished card facilitates understanding of their reasoning and fosters the ability to communicate this reasoning mathematically.

Note that students may not realise that overuse of high frequency numbers is not beneficial (for example, don't make a card full of '11's).

Bingo Card Blanks

BINGO				

COMMENTS

BINGO				

COMMENTS

BINGO				

COMMENTS

BINGO				

COMMENTS

Numbers for Calling Bingo

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

Addition Table

+	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

+	1	2	3	4	5	6	7	8	9	10
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										

Addition Table (Filled)

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

Addition Calling Cards

1 + 1	1 + 2	1 + 3	1 + 4	1 + 5
1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2 + 1	2 + 2	2 + 3	2 + 4	2 + 5
2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3 + 1	3 + 2	3 + 3	3 + 4	3 + 5
3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4 + 1	4 + 2	4 + 3	4 + 4	4 + 5
4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5 + 1	5 + 2	5 + 3	5 + 4	5 + 5
5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6 + 1	6 + 2	6 + 3	6 + 4	6 + 5
6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7 + 1	7 + 2	7 + 3	7 + 4	7 + 5
7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8 + 1	8 + 2	8 + 3	8 + 4	8 + 5
8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9 + 1	9 + 2	9 + 3	9 + 4	9 + 5
9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10 + 1	10 + 2	10 + 3	10 + 4	10 + 5
10 + 6	10 + 7	10 + 8	10 + 9	10 + 10

Likely Occurrence of Addition Facts

➔ Frequency of occurrence in every 100 calls, by number:

Frequency of occurrence	0	1	2	3	4	5	6	7	8	9	10
Number	0, 1, >20	2, 20	3, 19	4, 18	5, 17	6, 16	7, 15	8, 14	9, 13	10, 12	11

➔ Probability of occurrence each number, in every 100 calls, by number:

Probabilities are always numbers between 0 and 1. For older children, you can write these as probabilities, fractions or decimal fractions (4/100 or 0.04), or the words '4 out of 100' will make more sense to younger students.

Number	1	2	3	4	5	6	7	8	9	10
Probability of occurrence	0	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{3}{100}$	$\frac{4}{100}$	$\frac{5}{100}$	$\frac{6}{100}$	$\frac{7}{100}$	$\frac{8}{100}$	$\frac{9}{100}$
Probability of occurrence (decimal)	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09

Number	11	12	13	14	15	16	17	18	19	20
Probability of occurrence	$\frac{10}{100}$	$\frac{9}{100}$	$\frac{8}{100}$	$\frac{7}{100}$	$\frac{6}{100}$	$\frac{5}{100}$	$\frac{4}{100}$	$\frac{3}{100}$	$\frac{2}{100}$	$\frac{1}{100}$
Probability of occurrence (decimal)	0.10	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01